Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Wednesday 15 January 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. The table below shows corresponding values of x and y for $y = \log_2(2x)$

The values of y are given to 2 decimal places as appropriate.

x	2	5	8	11	14
y	2	3.32	4	4.46	4.81

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for $\int_2^{14} \log_2(2x)\,dx$, giving your answer to one decimal place. **(3)**

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_2^{14} \frac{\log_2(4x^2)}{5}\,dx$

(ii) $\int_2^{14} \log_2\left(\frac{2}{x}\right)dx$

(4)

a

TRAPEZIUM RULE

$\int_a^b y\,dx = \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\right]$, where $h = \frac{b-a}{n}$

represents the width of each strip

height of the first and last strip respectively

heights of strips in between the first and last

number of strips

$h = \frac{14-2}{4}$

There are 5 x values, so there are 5-1=4 strips

$h = 3$ ✓ B1

$\int_2^{14} \log_2(2x)\,dx = \frac{1}{2} \times 3 \times \left[(2 + 4.81) + 2(3.32 + 4 + 4.46)\right]$ ✓M1

$= 45.6$

$\boxed{= 45.6}$ ✓A1

bi. We just need to rewrite $\frac{\log_2(4x^2)}{5}$ so it is in terms of $\log_2(2x)$

①

$\boxed{\log(a^b) = b\log(a)}$

$\int_2^{14} \frac{\log_2(4x^2)}{5}\,dx = \int_2^{14} \frac{1}{5}\log_2\left[(2x)^2\right]dx$

$= \int_2^{14} \frac{1}{5} \times 2\log_2(2x)\,dx$ ① ✓M1

$= \int_2^{14} \frac{2}{5}\log_2(2x)\,dx$

$= \frac{2}{5}\int_2^{14} \log_2(2x)\,dx$

$= \frac{2}{5} \times 45.6$

$\boxed{= 18.2}$ ✓A1 ft

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Question 1 continued

bii. Like above, rewrite $\log_2\left(\frac{2}{x}\right)$ so it is in terms of $\log_2(2x)$

2

$$\boxed{\log\left(\frac{a}{b}\right) = \log(a) - \log(b)}$$

$$\int_2^{14} \log_2\left(\frac{2}{x}\right) dx = \int_2^{14} \log_2\left(\frac{4}{2x}\right) dx$$

$$= \int_2^{14} \log_2(4) - \log_2(2x)\, dx \quad ②$$

$$= \int_2^{14} 2 - \log_2(2x)\, dx \quad \checkmark \text{M1}$$

$$= \int_2^{14} 2\, dx - \int_2^{14} \log_2 2x\, dx$$

$$= \left[2x\right]_2^{14} - 45.6$$

$$= 2(14) - 2(2) - 45.6$$

$$= 28 - 4 - 45.6$$

$$\boxed{= -21.6} \quad \checkmark \text{A1ft}$$

(Total 7 marks)

Q1

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2. One of the terms in the binomial expansion of $(3 + ax)^6$, where a is a constant, is $540x^4$

(a) Find the possible values of a. **(4)**

(b) Hence find the term independent of x in the expansion of

$$\left(\frac{1}{81} + \frac{1}{x^6}\right)(3 + ax)^6$$

(3)

a. BINOMIAL EXPANSION FORMULA:

$(a + bx)^n = a^n + ({}^nC_1)a^{n-1}(bx)^1 + ({}^nC_2)a^{n-2}(bx)^2 + \dots + ({}^nC_{n-1})a^1(bx)^{n-1} + (bx)^n$

where ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

From the formula above, the fourth term will follow the form $({}^6C_4)(3)^{6-4}(ax)^4$ which is what the term $540x^4$ matches with.

$540x^4 = {}^6C_4 \times (3)^{6-4}(ax)^4$ ✓M1

$540x^4 = 15 \times 3^2 \times a^4x^4$ ✓A1

$540 = 135a^4$ ✓dM1

$a^4 = 4$

$a = \pm\sqrt{2}$

$\boxed{a = \pm\sqrt{2}}$ ✓A1

b. Independent of x refers to the constant term in the expansion

$\left(\frac{1}{81} + \frac{1}{x^6}\right)(3 + ax)^6 \xrightarrow{\text{constant term is}} \left(\frac{1}{81} + \frac{1}{x^6}\right)(\text{constant term} + \dots + x^6 \text{ term})$

The x^6 will cancel out

Constant term: $3^6 = 729$

x^6 term: $(ax)^6 = a^6x^6$

$= (\pm\sqrt{2})^6x^6$

$= 8x^6$

$\therefore$ constant term: $\frac{1}{81}(729) + \frac{1}{x^6}(8x^6) = 9 + 8$ ✓M1 ✓A1

$\boxed{= 17}$ ✓A1

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Question 2 continued

Q2

(Total 7 marks)

3. $$f(x) = 6x^3 + 17x^2 + 4x - 12$$

(a) Use the factor theorem to show that $(2x + 3)$ is a factor of $f(x)$.
(2)

(b) Hence, using algebra, write $f(x)$ as a product of three linear factors.
(4)

(c) Solve, for $\frac{\pi}{2} < \theta < \pi$, the equation

$$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 0$$

giving your answers to 3 significant figures.
(2)

a. FACTOR THEOREM: $(ax-b)$ is a factor of $f(x)$ if $f\left(\frac{b}{a}\right) = 0$ $\left(ax-b=0 \Rightarrow x = \frac{b}{a}\right)$

$2x+3=0$

$x = -\frac{3}{2}$

$f\left(-\frac{3}{2}\right) = 6\left(-\frac{3}{2}\right)^3 + 17\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) - 12$ ✓M1

$= -20.25 + 38.25 - 6 - 12$

$= 0$

$\therefore$ $(2x+3)$ is a factor of $f(x)$ ✓A1*

b.
$$\begin{array}{r|l} & 3x^2 + 4x - 4 \\ \hline 2x+3 & 6x^3 + 17x^2 + 4x - 12 \\ & -(6x^3 + 9x^2) \\ & \quad 8x^2 + 4x \\ & \quad -(8x^2 + 12x) \\ & \quad\quad -8x - 12 \\ & \quad\quad -(-8x - 12) \\ & \quad\quad 0 \quad 0 \end{array}$$
✓M1

$\therefore f(x) = (2x+3)(3x^2 + 4x - 4)$ ✓A1

$= (2x+3)(3x-2)(x+2)$ ✓dM1 ✓M1

c. let $y = \tan\theta$

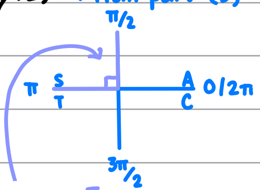
$6\tan^3\theta + 17\tan^2\theta + 4\tan\theta - 12 = 6y^3 + 17y^2 + 4y - 12$

$= (2y+3)(3y-2)(y+2)$ ← From part (b)

$2y+3=0$ $\quad$ $3y-2=0$ $\quad$ $y+2=0$

$y = -\frac{3}{2}$ $\quad$ $y = \frac{2}{3}$ $\quad$ $y = -2$

$\tan\theta = -\frac{3}{2}$ $\quad$ $\tan\theta = \frac{2}{3}$ $\quad$ $\tan\theta = -2$ ✓M1



At the range $\frac{\pi}{2} < \theta < \pi$, $\tan\theta$ is negative, so solve for $\tan\theta = -2$ and $\tan\theta = -\frac{3}{2}$

$\tan\theta = -\frac{3}{2}$ $\quad$ $\tan\theta = -2$

$\theta = \pi - \tan^{-1}\left(\frac{3}{2}\right)$ $\quad$ $\theta = \pi - \tan^{-1}(2)$

$\theta = 2.16$ $\quad$ $\theta = 2.03$ $\quad$ $\theta = 2.03, 2.16$ ✓A1

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Question 3 continued

Q3

(Total 8 marks)

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4.

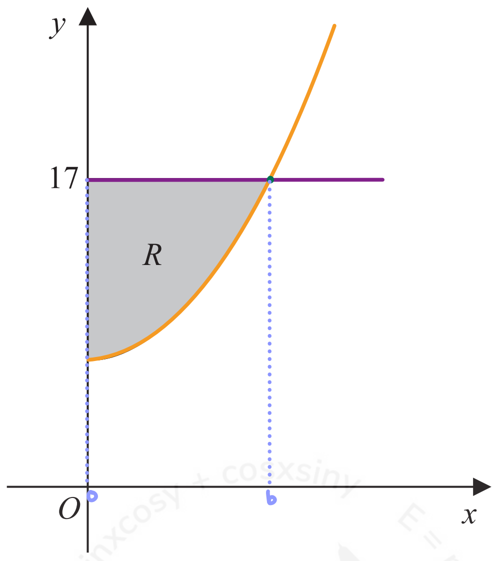

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = 2x^2 + 7 \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis and the line with equation $y = 17$

Find the exact area of R.

(6)

FORMULA FOR AREA BETWEEN TWO CURVES

area $= \int_a^b y_{TOP} - y_{BOTTOM} \, dx$

We can see that between $x=0$ and $x=b$ (limits which R is bounded by), y_{TOP} is $y=17$ and y_{BOTTOM} is $y=2x^2+7$

Finding b

$2x^2+7=17$

$2x^2=10$

$x^2=5$

$x=\pm\sqrt{5}$

$b=\sqrt{5}$ ✓B1

$\therefore$ area of $R = \int_0^{\sqrt{5}} 17-(2x^2+7) \, dx$

$= \int_0^{\sqrt{5}} 10-2x^2 \, dx$ ✓M1

$= \left[10x - \frac{2}{3}x^3\right]_0^{\sqrt{5}}$ ✓M1 ✓A1

$= 10(\sqrt{5}) - \frac{2}{3}(\sqrt{5})^3 - 0$ ✓M1

$= 10\sqrt{5} - \frac{10}{3}\sqrt{5}$

$= \boxed{\frac{20}{3}\sqrt{5}}$ ✓ A1

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Question 4 continued

Q4

(Total 6 marks)

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5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A colony of bees is being studied.

The number of bees in the colony at the start of the study was 30 000

Three years after the start of the study, the number of bees in the colony is 34 000

A model predicts that the number of bees in the colony will increase by $p\%$ each year, so that the number of bees in the colony at the end of each year of study forms a geometric sequence.

Assuming the model,

(a) find the value of p, giving your answer to 2 decimal places.

(3)

According to the model, at the end of N years of study the number of bees in the colony exceeds 75 000

(b) Find, showing all steps in your working, the smallest integer value of N.

(5)

a. NTH TERM OF GEOMETRIC SEQUENCE

$a_n = ar^{n-1}$

Start of study: 3rd year (4th term):

$a = 30000$

$a_3 = ar^{4-1}$

$34000 = 30000r^3$ ✓M1

$r^3 = \frac{17}{15}$

$r = 1.0426..$ ✓A1

$p = 4.26\%$ (2dp) ✓A1ft

b. Using the value of a and r in part (a):

$30000(1.0426)^N > 75000$ ✓M1

$(1.0426)^N > 2.5$ ✓A1

$N > \log_{1.0426}(2.5)$ ✓M1

$N > 21.96...$ ✓A1

smallest integer value of N = 22 ✓B1

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Question 5 continued

Q5

(Total 8 marks)

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6. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.

Given that the length of this chord is 4 units,

(c) find the value of p.

(3)

ai We can complete the square of circle C to find its centre and radius

$x^2 + y^2 + 6x - 4y - 14 = 0$

$x^2 + 6x + y^2 - 4y - 14 = 0$

$(x+3)^2 - 9 + (y-2)^2 - 4 - 14 = 0$

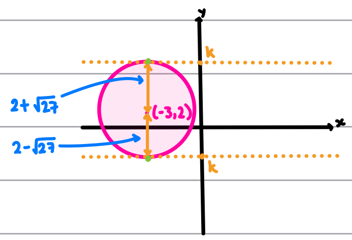
$(x+3)^2 + (y-2)^2 - 27 = 0$ ✓M1

$(x+3)^2 + (y-2)^2 = 27$

Centre: $x = -3$ $y = 2$ $\Rightarrow$ $\boxed{(-3,2)}$ ✓A1

aii. $\boxed{\text{Radius: } \sqrt{27}}$ ✓A1

b Tangent to C means only 1 intersection, and since $y=k$ is a vertical line, we can just add and subtract the radius of C from the y-coordinate of its centre



$\therefore k = 2 \pm \sqrt{27}$ ✓M1

$\boxed{k = 2 + 2\sqrt{3}}$, $\boxed{k = 2 - 2\sqrt{3}}$

✓A1 ✓A1

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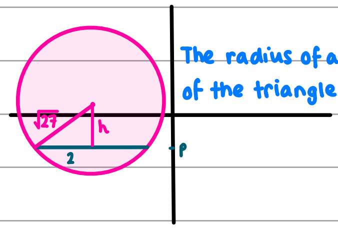
Question 6 continued

c.

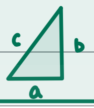


The radius of a circle bisects a chord if drawn perpendicularly to it, so the length of the base of the triangle is $4 \div 2 = 2$

Use pythagoras to find the height, h, of the triangle

PYTHAGORAS THEOREM

$a^2 + b^2 = c^2$



$a^2 + b^2 = c^2$

$h^2 + 2^2 = (\sqrt{27})^2$ ✓M1

$h^2 = 27 - 4$

$h^2 = 23$

$h = \sqrt{23}$

From the diagram, p = y-coordinate of centre of p − height of triangle

$p = 2 - \sqrt{23}$

$\boxed{p = 2 - \sqrt{23}}$ ✓dM1 ✓A1

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Question 6 continued

P 6 0 5 7 1 R A 0 1 4 2 8

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Question 6 continued

Q6

(Total 8 marks)

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7. (a) Show that the equation

$$8\tan\theta = 3\cos\theta$$

may be rewritten in the form

$$3\sin^2\theta + 8\sin\theta - 3 = 0$$

(3)

(b) Hence solve, for $0 \leqslant x \leqslant 90°$, the equation

$$8\tan 2x = 3\cos 2x$$

giving your answers to 2 decimal places.

(4)

a ① $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ ② $\boxed{\cos^2\theta = 1 - \sin^2\theta}$

$8\tan\theta = 3\cos\theta$

① $8\frac{\sin\theta}{\cos\theta} = 3\cos\theta$ ✓M1

$8\sin\theta = 3\cos^2\theta$

$8\sin\theta = 3(1-\sin^2\theta)$ ② ✓M1

$8\sin\theta = 3 - 3\sin^2\theta$

$3\sin^2\theta + 8\sin\theta - 3 = 0$

$\boxed{3\sin^2\theta + 8\sin\theta - 3 = 0}$ ✓A1

b Using the result obtained in part a.

$8\tan 2x = 3\cos 2x$

$3\sin^2 2x + 8\sin 2x - 3 = 0$ Factorise

$(3\sin 2x - 1)(\sin 2x + 3) = 0$ ✓M1

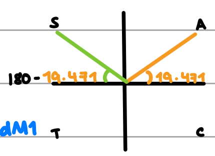
$\sin 2x + 3 = 0$

$\sin 2x = -3$ ← Ignore as sine has range $-1 \leq \sin x \leq 1$

$3\sin 2x - 1 = 0$

A1✓ $\sin 2x = \frac{1}{3}$ principal solution

$2x = 19.471, 180 - 19.471$ ✓dM1



Since range is $0 \leq x \leq 90°$, range of $2x$ is $0 \leq 2x \leq 180°$.
Also, since $\sin 2x$ is +ve, select 1st and 2nd quadrant of CAST diagram.

$2x = 19.471°, 160.529°$

$\boxed{x = 9.74°, 80.26° \text{ (2dp)}}$ ✓M1

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Question 7 continued

Q7

(Total 7 marks)

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8. (i) An arithmetic series has first term a and common difference d.

Prove that the sum to n terms of this series is

$$\frac{n}{2}\{2a+(n-1)d\}$$

(3)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_n = 5n + 3(-1)^n$$

Find the value of

(a) u_5

(1)

(b) $\sum_{n=1}^{59} u_n$

(3)

i. We know an arithmetic sequence has a first term, a, and a common difference, d

1st term: a

2nd term: $a + d$

(...)

$(n-1)$th term: $a + (n-2)d$

nth term: $a + (n-1)d$

So the sum of an arithmetic series is:

✓B1

$S = a + a + d + \ldots + a + (n-2)d + a + (n-1)d$ Write in reverse

$S = a + (n-1)d + a + (n-2)d + \ldots + a + d + a$ ✓M1

Add the two series together (you can see how each term matches and adds up)

$a + a + (n-1)d = 2a + (n-1)d$

$a + d + a + (n-2)d = 2a + (n-1)d$

The addition above repeats for all the n terms, resulting in:

$2S = n(2a + (n-1)d)$

$\boxed{S = \frac{n}{2}(2a + (n-1)d)}$ ✓A1*

IIa. Sub in $n=5$ to u_n

$u_n = 5n + 3(-1)^n$

$u_n = 5(5) + 3(-1)^5$

$\boxed{u_n = 22}$ ✓A1

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Question 8 continued

iib.

① $\boxed{\sum_{r=k}^{n} f(r)+g(r) = \sum_{r=k}^{n} f(r) + \sum_{r=k}^{n} g(r)}$

② $\boxed{\sum_{r=k}^{n} m f(r) = m\sum_{r=k}^{n} f(r)}$

$$\sum_{n=1}^{59} U_n = \sum_{n=1}^{59} 5n + 3(-1)^n$$

$$= \sum_{n=1}^{59} 5n + \sum_{n=1}^{59} 3(-1)^n \quad ①$$

$$= 5\sum_{n=1}^{59} n + \sum_{n=1}^{59} 3(-1)^n \quad ②$$

This is just an arithmetic series with first term 1 and last term 59.

You can see that when n is odd, $3(-1)^n = -3$, and when n is even, $3(-1)^n = 3$. This means when n is even, $\sum_{n=1}^{n} 3(-1)^n = 0$, as the 3 and -3 cancels out, so when n is odd, $\sum_{n=1}^{n} 3(-1)^n = -3$, as the 3 isn't there to cancel out the -3

ARITHMETIC SERIES:

$S_n = \frac{n}{2}(a + l)$

l: last term

$\sum_{n=1}^{n} 3(-1)^n = -3$ ✓B1

$$5\sum_{n=1}^{59} n = 5\left[\frac{59}{2}(1+59)\right] \text{ ✓M1}$$

$$= 5 \times 1770$$

$$= 8850$$

$\therefore 8850 - 3 = \boxed{8847}$ ✓A1

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Question 8 continued

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Question 8 continued

Q8

(Total 7 marks)

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9. (a) Sketch the curve with equation

$$y = 3 \times 4^x$$

showing the coordinates of any points of intersection with the coordinate axes.

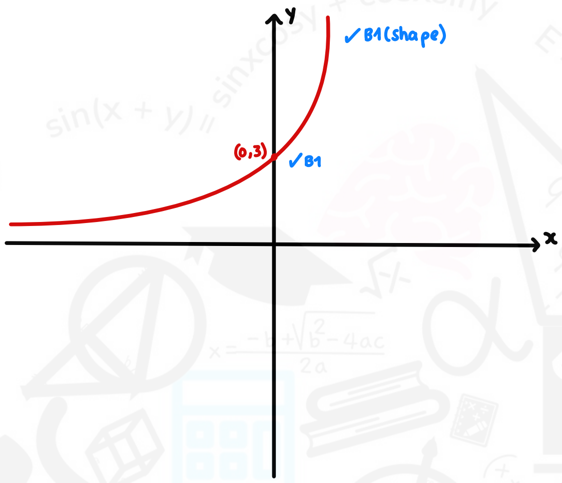
(2)

The curve with equation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

(b) Show that the x coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

a.



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Question 9 continued

b.

① $\boxed{\log(a^b) = b\log(a)}$ ② $\boxed{\log(ab) = \log a + \log b}$ ③ $\boxed{\log\left(\frac{a}{b}\right) = \log(a) - \log(b)}$

$3\times 4^x = 6^{1-x}$

$\log(3\times 4^x) = \log(6^{1-x})$ ✓M1

② $\log 3 + \log(4^x) = \log(6^{1-x})$

$\log 3 + x\log 4 = (1-x)\log 6$ ① ✓dM1

$\log 3 + x\log 4 = \log 6 - x\log 6$

$x\log 4 + x\log 6 = \log 6 - \log 3$ ✓A1

② $x\log(4\times 6) = \log\left(\frac{6}{3}\right)$ ③ ✓ddM1

$x\log(24) = \log(2)$

$\boxed{x = \frac{\log(2)}{\log(24)}}$ ✓A1*

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Question 9 continued

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Question 9 continued

Q9

(Total 7 marks)

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10. A curve C has equation

$$y = 4x^3 - 9x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P with x coordinate $\frac{1}{2}$ lies on C.

Given that P is a stationary point of C,

(a) show that $k = -\frac{3}{2}$ **(4)**

(b) Determine the nature of the stationary point at P, justifying your answer. **(2)**

The curve C has a second stationary point.

(c) Using algebra, find the x coordinate of this second stationary point. **(4)**

a. $\frac{dy}{dx} = 0$ at a stationary point.

$y = 4x^3 - 9x + \frac{k}{x}$

$\frac{dy}{dx} = 12x^2 - 9 - \frac{k}{x^2}$ ✓M1 ✓A1

Sub in P with x-coordinate $\frac{1}{2}$

$12\left(\frac{1}{2}\right)^2 - 9 - \frac{k}{\left(\frac{1}{2}\right)^2} = 0$ ✓dM1

$3 - 9 - 4k = 0$

$4k = -6$

$\boxed{k = -\frac{3}{2}}$ ✓A1*

b $\boxed{\text{MINIMUM: } \frac{d^2y}{dx^2} > 0 \quad \text{MAXIMUM: } \frac{d^2y}{dx^2} < 0}$

Using the value of k from part a

$\frac{dy}{dx} = 12x^2 - 9 + \frac{3}{2x^2}$

$\frac{d^2y}{dx^2} = 24x - \frac{3}{x^3}$ ✓M1

Subbing in $x = \frac{1}{2}$

$\frac{d^2y}{dx^2} = 24\left(\frac{1}{2}\right) - \frac{3}{\left(\frac{1}{2}\right)^3}$

Leave blank

Question 10 continued

$\frac{d^2y}{dx^2} = -12$

$\boxed{\text{Since } \frac{d^2y}{dx^2} = -12 < 0 \text{, P is a maximum}}$ ✓A1

c. We need to find the other solution to $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

$12x^2 - 9 + \frac{3}{2x^2} = 0$ Multiply by $2x^2$

A1✓ $24x^4 - 18x^2 + 3 = 0$ Factorise

$3(4x^2-1)\underbrace{(2x^2-1)} = 0$ ✓A1

$2x^2 - 1 = 0$

✓dM1

$x^2 = \frac{1}{2}$ Ignore -ve solution as $x>0$ in the question.

$\boxed{x = \frac{\sqrt{2}}{2}}$ ✓A1

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Question 10 continued

Q10

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END